CHAPTER 5. USING THE INFORMATION GIVEN BY THE DERIVATIVE

Section 5.1 Increasing and Decreasing Functions

Quick Quiz:
1. See page 276.
2. Zeroes of $f$: $f(x) = 0 \iff (x = 0 \text{ or } x = 1)$. Choose the test points $-1, \frac{1}{2}, 1$. The information is summarized below.

   \[ \begin{align*}
   \text{Sign of } f'(x) &:\quad (\times)(-) > 0 \quad \circ \quad (+)(-) < 0 \quad 1 \quad (+)(+) > 0 \\
   \end{align*} \]

3. TRUE
4. TRUE

END-OF-SECTION EXERCISES:
1. Positive: $(-\infty, -2) \cup (1, \infty)$
   Negative: $(-2, 1)$
3. Positive: $(-\infty, -1) \cup (3, \infty)$
   Negative: $(-1, 3)$
5. Positive: $(-\infty, \frac{1}{3}) \cup (\frac{3}{4}, \infty)$
   Negative: $(-\frac{1}{3}, \frac{3}{4})$
7. Positive: $(0, \infty)$
   Negative: $(-\infty, -1) \cup (-1, 0)$
9. Positive: $(-4, -1) \cup (-1, 0)$
   Negative: $(-\infty, -4) \cup (-1, 1)$
11. Positive: $(0, \infty)$
    Negative: $(-\infty, 0)$
13. Positive: $(1, \infty)$
    Negative: $(\frac{1}{2}, 1)$
15. The function $f$ increases on $(-\infty, -2) \cup (1, \infty)$ and decreases on $(-2, 1)$.
17. The function $f$ decreases on $(-\infty, -1)$ and increases on $(-1, \infty)$.
19. The function $f$ decreases on $(0, \frac{1}{3})$, and increases on $(\frac{1}{3}, \infty)$.

21. b) 2278
c) 3870
23. c) $1 + 2 + 2^2 + 2^3 + 2^4 = 31$
d) $2^5 + \cdots + 2^{10} = 1984$

Section 5.2 Local Maxima and Minima—Critical Points

Quick Quiz:
1. The point $(c, f(c))$ must be a critical point. Thus, either it is an endpoint of the domain of $f$, or $f'(c) = 0$, or $f'(c)$ does not exist.
2. NO! There are critical points that are not local extreme points.
3. The ‘critical points’ for a function $f$ are the CANDIDATES for the local extreme points of $f$.
4. NO! When $A \Rightarrow B$ is true, $B \Rightarrow A$ may be either true or false.
5. Since $f$ is differentiable, it is also continuous. By the First Derivative Test, there is a maximum at $x = a$; a minimum at $x = c_1$; a maximum at $x = c_2$; and a minimum at $x = b$.

END-OF-SECTION EXERCISES:
3. TRUE
5. TRUE
Section 5.3 The Second Derivative—Inflection Points

Quick Quiz:
1. The second derivative of a function tells us the rate of change of the slopes of the tangent lines. This information is referred to as the *concavity* of the function.

2. If $f$ is concave up on $I$ if and only if $f''(x) > 0$ for every $x \in I$.

3. The sentence is false. Choose $x$ to be $-1$. Then the hypothesis ‘$(-1)^2 = 1$’ is true, but the conclusion ‘$-1 = 1$’ is false.

4. By the Second Derivative Test, the point $(c, f(c))$ is a local maximum point for $f$.

5. $f'(x) = 3(x-1)^2$, $f''(x) = 6(x-1)$, so $f''(1) = 6(1-1) = 0$

END-OF-SECTION EXERCISES:
1. local minima at $x = 0$ and $x = 1$; local maximum at $x = \frac{1}{2}$
2. $f(x)$ is positive on $(-\infty, -2.5) \cup (-2, \infty)$
   $f(x)$ is negative on $(-2.5, -2)$
3. $f$ is concave up on $(-2, 2)$
   $f$ is concave down on $(-3, -2) \cup (2, \infty)$
4. $D(f') = \mathbb{R} - \{-4, -3, -2\}$
5. $\{x \mid f(x) > 10\} = (-2, -1.5)$
6. $\{x \mid f''(x) < 0\} = (-3, -2) \cup (2, \infty)$
7. $\{x \mid f''(x) > 0\}$ does not exist
8. The critical points are: $\{(x, 4) \mid x \in (-\infty, -4)\}$, $(0, 2)$, $(-4, 4)$ and $(-3, 8)$
9. $\{x \in D(f) \mid f$ is not differentiable at $x\} = \{-4, -3\}$
10. $\lim_{h \to 0} \frac{f(0+h)-f(0)}{h} = f'(0) = 0$

Section 5.4 Graphing Functions—Some Basic Techniques

Quick Quiz:
1. 
2. For $x \gg 0$ and $x \ll 0$, $P(x) \approx -6x^7$. So as $x \to \infty$, $P(x) \to -\infty$.
   As $x \to -\infty$, $P(x) \to \infty$.
3. $f(-x) = (-x)^5 = (-x) = -x^5 + x = -(x^5 - x) = -f(x)$. Thus, $f$ is ODD, but not EVEN.
4. $f'(x) = 12x - 7$; $f'(x) = 0 \iff x = \frac{7}{12}$
   There is a horizontal tangent line at $(\frac{7}{12}, f(\frac{7}{12}))$; $f(\frac{7}{12}) = 6(\frac{7}{12})^2 - 7(\frac{7}{12}) - 3 \approx -5.04$
   $f''(x) = 12$, so $f''(x) > 0$ for all $x$
Section 5.5  More Graphing Techniques

Quick Quiz:
1. Find $A$ and $B$ for which $AB = (3)(-8) = -24$ and $A + B = -2$; take $A = -6$ and $B = 4$. Then:

$$3x^2 - 2x - 8 = 3x^2 - 6x + 4x - 8$$
$$= 3x(x - 2) + 4(x - 2)$$
$$= (3x + 4)(x - 2)$$

2. First, solve $3x^2 - 2x - 8 = 0$ using the Quadratic Formula:

$$x = \frac{2 \pm \sqrt{4 - 4(3)(-8)}}{6} = \frac{2 \pm 10}{6} = \frac{2 + 4}{3}, \frac{2 - 4}{3}$$

Then:

$$3x^2 - 2x - 8 = 3(x - 2)(x + \frac{4}{3})$$
$$= (x - 2)(3x + 4)$$

3. CANDIDATES: $\pm \frac{1 \pm 2}{3} = \pm 1, \pm 2$

4. $\text{not}(A \text{ and } B) \iff (\text{not } A) \text{ or } (\text{not } B)$

5. $P(1) = -1$; the remainder upon division by $x - 1$ equals $-1$

END-OF-SECTION EXERCISES:
1. $P(x) = 2x^3 - 3x^2 - 3x - 5 = (x^2 + x + 1)(2x - 5)$
3. $P(x) = x^4 - 5x^2 + 6 = (x - \sqrt{2})(x + \sqrt{2})(x - \sqrt{3})(x + \sqrt{3})$

Section 5.6  Asymptotes—Checking Behavior at Infinity

Quick Quiz:
1. An asymptote is a curve (often a line) that a graph gets close to as $x$ approaches $\pm \infty$, or some finite number.

2. $\lim_{x \to c^-} f(x) = -\infty \iff \forall M < 0 \ \exists \delta > 0$ such that if $x \in (c - \delta, c)$, then $f(x) < M$

3. VERTICAL: $x = -2$
   HORIZONTAL: $y = 3$

4. Both individual limits (the ‘numerator’ limit and the ‘denominator’ limit) must exist. Also, the ‘denominator’ limit cannot equal zero.