

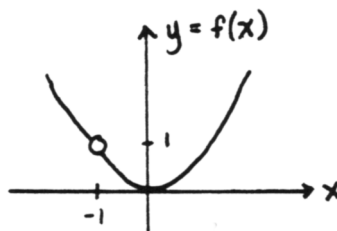
CHAPTER 3. LIMITS AND CONTINUITY

Section 3.1 Limits—The Idea

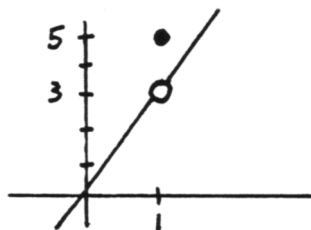
Quick Quiz:

1. $\lim_{x \rightarrow -2} x^3 = (-2)^3 = -8$

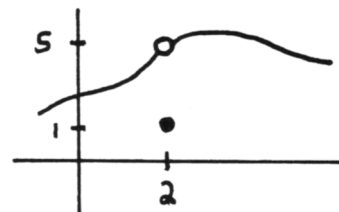
2. $\lim_{x \rightarrow -1} f(x) = (-1)^2 = 1$



3. $\lim_{x \rightarrow 1} f(x) = 3(1) = 3$



4. There are many correct graphs. The graph must contain the point $(2, 1)$; and when the inputs are close to 2 (but not equal to 2), the outputs must be close to 5.



5. $|t - (-1)| \leq 4$

End-of-Section Exercises:

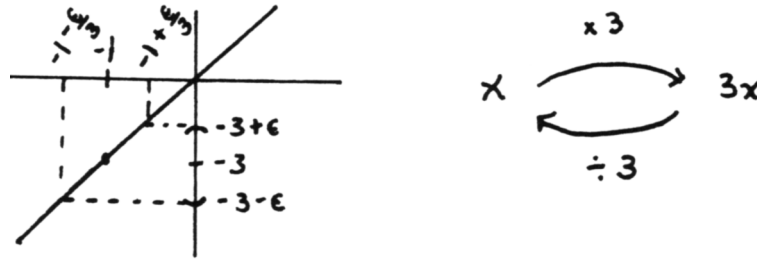
1. EXP
3. SEN; T
5. SEN; C
7. SEN; T
9. SEN; C
11. EXP
13. SEN; (always) T
15. SEN; (always) T
17. SEN; C
19. SEN; T

Section 3.2 Limits—Making It Precise

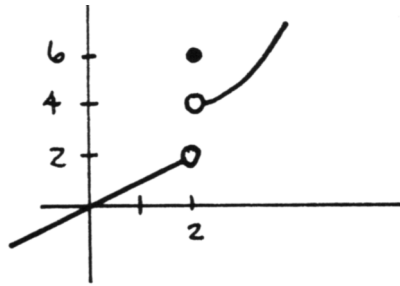
Quick Quiz:

1. $\lim_{x \rightarrow c} f(x) = l \iff \forall \epsilon > 0, \exists \delta > 0, \text{ such that if } 0 < |x - c| < \delta \text{ and } x \in \mathcal{D}(f), \text{ then } |f(x) - l| < \epsilon$

2. The 'four step process' is summarized using the mapping diagram and sketch given below. Given $\epsilon > 0$, take $\delta = \frac{\epsilon}{3}$.

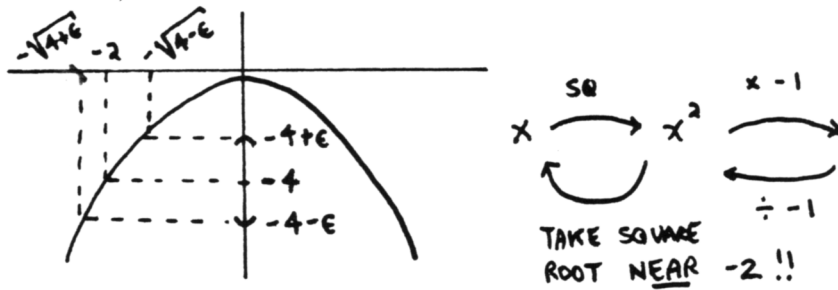


3. $\lim_{x \rightarrow 2} f(x)$ does not exist
 $\lim_{x \rightarrow 2^+} f(x) = 4$
 $\lim_{x \rightarrow 2^-} f(x) = 2$

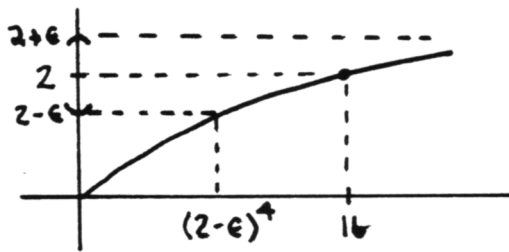


End-of-Section Exercises:

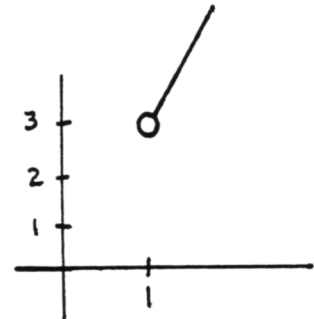
1. When 'undoing' the output $-4 - \epsilon$, it is important to take the input that lies near -2 ! Take: $\delta := -2 + \sqrt{4 + \epsilon}$



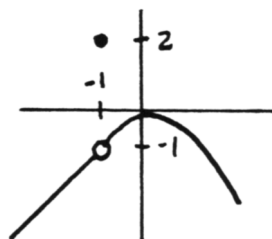
3. Take $\delta := 16 - (2 - \epsilon)^4$, since this is the shorter distance.



5. $\lim_{x \rightarrow 1} f(x) = 3$
 $\lim_{x \rightarrow 1^+} f(x) = 3$
 $\lim_{x \rightarrow 1^-} f(x)$ is not defined, since f is not defined to the left of 1



7. $\lim_{x \rightarrow -1} g(x) = -1$
 $\lim_{x \rightarrow -1^+} g(x) = -1$
 $\lim_{x \rightarrow -1^-} g(x) = -1$



End-of-Section Exercises:

1. SEN; CONDITIONAL
3. EXP
5. SEN; CONDITIONAL
7. SEN; CONDITIONAL
9. SEN; CONDITIONAL
11. SEN; TRUE
13. SEN; FALSE
15. EXP. Out of context, it is not known if this is a POINT (a, b) , or an open interval of real numbers. In either case, however, it is an EXPRESSION.

Section 3.5 Indeterminate Forms

Quick Quiz:

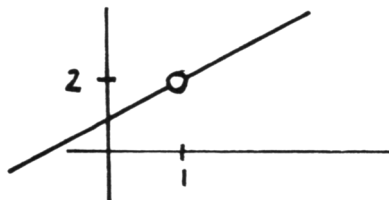
1. An 'indeterminate form' is a limit that, upon direct substitution, results in one of the forms: $\frac{0}{0}$, 1^∞ , or $\frac{\pm\infty}{\pm\infty}$.
- 2.

$$\begin{aligned}\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} &= \lim_{x \rightarrow 1} \frac{(x - 1)(x + 1)}{x - 1} \\ &= \lim_{x \rightarrow 1} (x + 1) = 1 + 1 = 2\end{aligned}$$

3. NO! It is true for all values of x except 1. When x is 1, the left-hand side is not defined; the right-hand side equals 2.
- 4.

$$y = \frac{x^2 - 1}{x - 1} = \frac{(x - 1)(x + 1)}{x - 1} \text{ for } x \neq 1 \quad x + 1$$

The graph is:



5. The graph of f is the same as the graph of $y = \frac{x^2 - 1}{x - 1}$. See (4).
6. $f = g$ if and only if $\mathcal{D}(f) = \mathcal{D}(g)$, and $f(x) = g(x)$ for all x in the common domain.

End-of-Section Exercises:

1. SEN; FALSE
3. SEN; FALSE
5. SEN; TRUE
7. SEN; TRUE. (Either both limits do not exist; or they both exist, and are equal.)
- 9.

$$\begin{aligned}\lim_{x \rightarrow -1} \frac{x^3 + x^2 - 3x - 3}{x + 1} &= \lim_{x \rightarrow -1} \frac{(x + 1)(x^2 - 3)}{x + 1} \\ &= \lim_{x \rightarrow -1} x^2 - 3 \\ &= (-1)^2 - 3 = -2\end{aligned}$$

11.
$$\lim_{x \rightarrow 2} \frac{x + 2}{x^2 + 4x + 4} = \frac{2 + 2}{2^2 + 4(2) + 4} = \frac{4}{16} = \frac{1}{4}$$

13. $\lim_{t \rightarrow 0^+} (1+t)^{1/t} = e$

Section 3.6 The Intermediate Value Theorem

Quick Quiz:

1. If f is continuous on $[a, b]$, and D is any number between $f(a)$ and $f(b)$, then there exists $c \in (a, b)$ with $f(c) = D$.
2. Since f is continuous on $[1, 3]$ and 0 is a number between $f(1)$ and $f(3)$, the Intermediate Value Theorem guarantees the existence of a number c with $f(c) = 0$.
3. TRUE. When the hypothesis of an implication is false, the implication is (vacuously) true.
4. FALSE. Let $x = -1$. Then the hypothesis $|-1| = 1$ is true, but the conclusion $-1 = 1$ is false.

5.

A	B	$A \Rightarrow B$
T	T	T
T	F	F
F	T	T
F	F	T

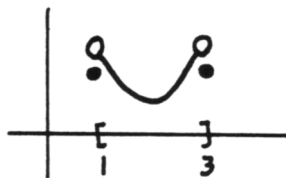
END-OF-SECTION EXERCISES:

1. TRUE
3. TRUE
5. TRUE
7. TRUE
9. TRUE

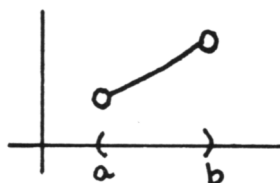
Section 3.7 The Max-Min Theorem

Quick Quiz:

1. The symbol ' \iff ' can also be read as 'if and only if'.
The number $f(c)$ is a maximum of f on I if and only if $f(x) \leq f(c)$ for all $x \in I$.
2. There are many possible correct graphs.



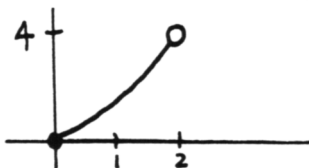
3. There are many possible correct graphs.



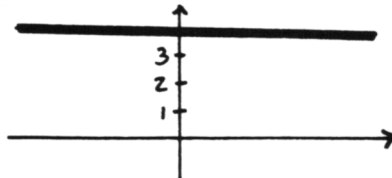
4. If f is continuous on $[a, b]$, then f attains both a maximum and minimum value on $[a, b]$.
5. The contrapositive of the sentence ' $A \implies B$ ' is the sentence ' $\text{not } B \implies \text{not } A$ '.
An implication is equivalent to its contrapositive. That is, the sentences ' $A \implies B$ ' and ' $\text{not } B \implies \text{not } A$ ' always have the same truth values, regardless of the truth values of A and B .

END-OF-SECTION EXERCISES:

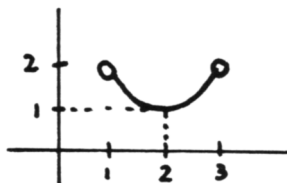
1. The minimum value of f on I is 0; there is no maximum value. The only minimum point is $(0, 0)$.



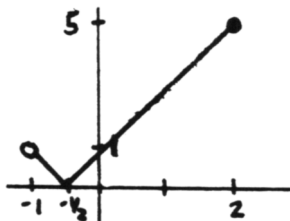
3. The maximum value of f on I is 4; the minimum value of f on I is 4. The points $(x, 4)$ for $x \in \mathbb{R}$ are all both maximum and minimum points.



5. The minimum value of f on I is 1; there is no maximum value. The point $(2, 1)$ is the only minimum point.



7. The minimum value of f on I is 0; the maximum value is 5. The only minimum point is $(-\frac{1}{2}, 0)$; the only maximum point is $(2, 5)$.

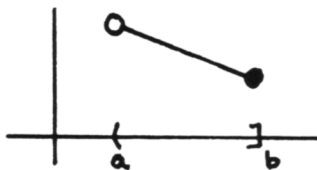


9. TRUE

Contrapositive: If f does not attain a maximum value on $[a, b]$, then f is not continuous on $[a, b]$.

11. FALSE

Counterexample: Let f be the function graphed below. Then, the hypothesis ' f is continuous on $(a, b]$ ' is TRUE, but the conclusion ' f attains a maximum value on $(a, b]$ ' is FALSE.



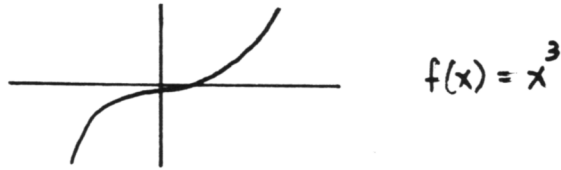
Contrapositive: If f does not attain a maximum value on $(a, b]$, then f is not continuous on $(a, b]$.

13. TRUE

Contrapositive: If f does NOT attain both a maximum and minimum value on $[1, 2]$, then f is not continuous on $(0, 5)$.

15. FALSE.

Counterexample: Let f be the function graphed below. Then the hypothesis ' f is continuous on \mathbb{R} ' is TRUE, but the conclusion ' f attains a maximum value on \mathbb{R} ' is FALSE.



Contrapositive: If f does not attain a maximum value on \mathbb{R} , then f is not continuous on \mathbb{R} .