ABBREVIATED SOLUTIONS TO
QUICK QUIZ QUESTIONS and ODD-NUMBERED END-OF-SECTION EXERCISES

CHAPTER 1. ESSENTIAL PRELIMINARIES

Section 1.1 The Language of Mathematics—Expressions versus Sentences

Quick Quiz:
1. a mathematical expression
2. numbers, functions, sets
3. \( x = \frac{x}{2} + \frac{x}{2} \) (many others are possible)
4. \( \sqrt{x} > 2 \) and \( 4 - 3 = 7 \) are sentences

End-of-Section Exercises:
1. EXP
2. SEN, T
3. SEN, T
4. SEN, F
5. SEN, T
6. EXP
7. SEN, T
8. EXP
9. SEN, ST/SF
10. SEN, T
11. EXP
12. SEN, T
13. EXP
14. SEN, T
15. EXP
16. SEN, ST/SF
17. SEN, ST/SF
18. EXP
19. SEN, T
20. SEN, F
21. SEN, F
22. SEN, T
23. SEN, T
24. SEN, ST/SF
25. SEN, ST/SF
26. EXP
27. EXP
28. SEN, T
29. SEN, T
30. SEN, T
31. SEN, T
32. SEN, T
33. SEN, ST/SF
34. SEN, T
35. SEN, F
36. Commutative Property of Addition
37. Distributive Property
38. If \( x = 1 \) and \( y = 3 \): \( 1 - 3 = 1 + (-3) = -2 \)
   If \( x = 1 \) and \( y = -3 \): \( 1 - (-3) = 1 + (-(-3)) = 1 + 3 = 4 \)
39. The expression \( xyz \) is not ambiguous; if one person computes this as \((xy)z\) and another as \(x(yz)\), the same results are obtained.

Section 1.2 The Role of Variables

Quick Quiz:
1. The variables are \( x \) and \( y \); the constants are \( A \), \( B \), and \( C \).
2. With universal set \( \mathbb{R} \), \( x^2 = 3 \) has solution set \( \{\sqrt{3}, -\sqrt{3}\} \). With universal set \( \mathbb{Z} \), the solution set is empty.
3. To 'solve' an equation means to find all choices (from some universal set) that make the equation true.
   Three solutions of \( x + y = 4 \): \((0, 4), (4, 0), \) and \((2, 2)\). There are an infinite number of solution pairs!
4. The equation \( x^2 \geq 0 \) is (always) true. The equation \( x > 0 \) is conditional; it is true for \( x \in (0, \infty) \), and false otherwise.
5. Choose two from the following list:
   - variables are used in mathematical expressions to denote quantities that are allowed to vary (like in the formula \( A = \pi r^2 \));
   - variables are used to denote a quantity that is initially unknown, but that one would like to know (for example, 'solve \( 2x + 3 = 5 \)');
   - variables are used to state a general principle (like the commutative law of addition).

End-of-Section Exercises:
1. EXP
2. SEN, T
3. SEN, T
4. SEN, F
5. SEN, F
6. SEN, T
7. SEN, F
8. SEN, T
9. EXP
10. SEN, T
11. EXP
12. SEN, T
13. EXP
14. SEN, T
15. EXP
16. SEN, ST/SF
17. SEN, ST/SF
18. EXP
19. SEN, T
20. SEN, F
21. SEN, F
22. SEN, T
23. SEN, T
24. SEN, ST/SF
25. SEN, ST/SF
26. EXP
27. EXP
28. SEN, T
29. SEN, T
30. SEN, T
31. SEN, T
32. SEN, T
33. SEN, ST/SF
34. SEN, T
35. SEN, F
36. Commutative Property of Addition
37. Distributive Property
38. If \( x = 1 \) and \( y = 3 \): \( 1 - 3 = 1 + (-3) = -2 \)
   If \( x = 1 \) and \( y = -3 \): \( 1 - (-3) = 1 + (-(-3)) = 1 + 3 = 4 \)
39. The expression \( xyz \) is not ambiguous; if one person computes this as \((xy)z\) and another as \(x(yz)\), the same results are obtained.
9. SEN, F  
11. SEN, T
13. EXP
15. \( \mathbb{R} \): the only solution is 1;  
   the rational numbers: the only solution is 1;  
   the integers: the only solution is 1.
17. \( \mathbb{R} \): setting each factor to 0, the real number solutions are 1, \(-\pi\), and \(\frac{3}{2}\);  
   the rational numbers: the only rational solutions are 1 and \(\frac{3}{2}\);  
   the integers: the only integer solution is 1.
19. a) The points are plotted at right:

\[
\begin{array}{c}
\text{point 1} \\
\text{point 2} \\
\text{point 3}
\end{array}
\]

b) Since \((-\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2 = \frac{1}{4} + \frac{3}{4} = 1\), the point \((-\frac{1}{2}, \frac{\sqrt{3}}{2})\) lies on the unit circle. Same for the remaining point.
c) Clearly, the number 1 satisfies the equation. To see that \(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\) satisfies \(x^3 = 1\), observe that:
\[
\begin{align*}
\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^3 &= \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) \\
&= \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) \\
&= \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) \\
&= \frac{1}{4} + \frac{3}{4} \\
&= 1
\end{align*}
\]
Similarly for the remaining number.
d) The equation \(x^3 - 1 = 0\) has the same solutions as the equation \(x^3 = 1\), so the problem has already been solved.

Section 1.3 Sets and Set Notation

Quick Quiz:
1. F; the set has only 3 members
2. F
3. T
4. F
5. 105 = 5 \cdot 3 \cdot 7; F

End-of-Section Exercises:
1. EXP; this is a set
3. SEN, T
5. SEN, F
7. SEN, C. The truth of this sentence depends upon the set \(S\) and the element \(x\).
9. SEN, C. The truth depends on \(x\). If \(x\) is 1, 2, or 3, then the sentence is true. Otherwise, it is false.
11. EXP; this is a set
13. SEN, T
15. SEN, C. The only number that makes this true is 1.
17. SEN, T. No matter what real number is chosen for \( x \), both component sentences ‘\(|x| \geq 0\)’ and ‘\(x^2 \geq 0\)’ are true.
19. SEN, T. The two elements are both sets: \( \{1\} \) and \( \{1, \{2\}\} \)
21. SEN, F. The number \( \frac{3}{7} \) is in reduced form; the denominator has factors other than 2’s and 5’s.

**Section 1.4 Mathematical Equivalence**

Quick Quiz:
1. F; when \( x = -2 \), the first sentence is false, but the second is true.
2. THEOREM: For all real numbers \( a \), \( b \) and \( c \):
   \[
   a = b \iff a + c = b + c
   \]
3. THEOREM: For all real numbers \( a \), \( b \) and \( c \):
   \[
   a > b \iff a + c > b + c
   \]
4. \( \{(x, y) \mid x \neq 3 \text{ and } y \neq 0\} \)
5. equivalent
6. expressions; sentences

End-of-Section Exercises:
3. SEN, T. Both sentences have the same implied domain, and the same solution set, \( \{4\} \).
5. SEN, T. Both sentences have the same implied domain, and the same solution set, \( \{0\} \).
7. EXP
9. SEN, T. Both sentences have the same implied domain, and the same solution set, \( \{-2\} \).
11.
   \[
   5x - 7 = 3 \iff 5x = 10 \quad \text{(add 7)}
   \iff x = 2 \quad \text{(divide by 5)}
   \]
13.
   \[
   3x < x - 11 \iff 2x < -11 \quad \text{(subtract } x) \\
   \iff x < -\frac{11}{2} \quad \text{(divide by the positive number } 2) \\
   \]

**Section 1.5 Graphs**

Quick Quiz:
1.
2. 

3. \( y - x^2 + 1 = 0 \iff y = x^2 - 1; \)

4. First graph the boundary, \( y = 2x \). We want all points on or below this line.

5. TRUE

6. 

End-of-Section Exercises:
1. \( x = \pi \)
3. \( |x| = 2 \iff x = 2 \) or \( x = -2 \)
5. \( 3x < -2 \iff x < -\frac{2}{3} \)
7. \( x = 0 \) or \( |x| = 1 \)
9. \( x = 1 \) or \( |x| = 1 \)

11. The critical observation here is that there are TWO numbers whose absolute value is 7: 7, and \(-7\). Thus:

\[
|3x + 1| = 7 \iff 3x + 1 = 7 \text{ or } 3x + 1 = -7
\iff 3x = 6 \text{ or } 3x = -8
\iff x = 2 \text{ or } x = -\frac{8}{3}
\]

The solution set is \( \{2, -\frac{8}{3}\} \).
13. \( x + y = 2 \iff y = -x + 2 \). The graph is the line that crosses the \( y \)-axis at 2, and has slope \(-1\).

15. The graph of \( x = 1 \) or \( y = -2 \) is the set of all points with \( x \)-coordinate 1, together with all points with \( y \)-coordinate \(-2\). The graph is shown below.

17. The solution set of \( |y| = 1 \), viewed as an equation in two variables, is \( \{(x, y) \mid x \in \mathbb{R}, |y| = 1\} \). Thus, we seek all points with \( y \)-coordinates 1 or \(-1\). See the graph below.

19.

\[
|x + y| = 1 \iff x + y = 1 \text{ or } x + y = -1 \iff y = -x + 1 \text{ or } y = -x - 1
\]

The graph is the two lines shown below.