

NAME \_\_\_\_\_

SAMPLE TEST, worth 100 points, Chapter 6

Show all work that leads to your answers. Good luck!

8 pts

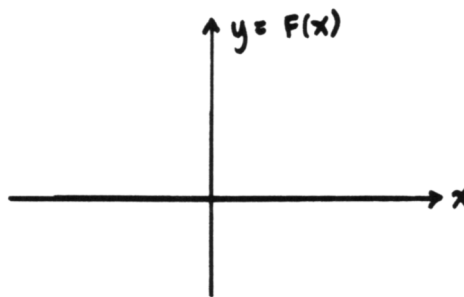
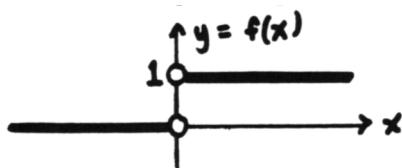
TRUE or FALSE. Circle the correct response. (2 points each)

- T F  $F(x) = x \ln x - x + 2$  is an antiderivative of  $f(x) = \ln x$ .
- T F If  $f'(x) = g'(x)$  for all  $x \in \mathbb{R}$ , then  $f$  and  $g$  differ by at most a constant.
- T F  $\int t^2 dt = t^3 + C$
- T F An antiderivative of  $f$  is a function with derivative  $f$ .

6 pts

The graph of a function  $f$  is given below. In the space provided, graph a function  $F$  satisfying:

- $F'(x) = f(x) \quad \forall x \in \mathcal{D}(f)$  ;
- $F$  is continuous at 0 ; and
- $F(-1) = 2$ .



13 pts

(6 pts) These questions have to do with the indefinite integral  $\int f(x) dx$ .

Fill in the blanks:

The symbol  $\int$  is called the \_\_\_\_\_.

The function  $f$  being integrated is called the \_\_\_\_\_.

The process of finding  $\int f(x) dx$  is called \_\_\_\_\_.

(There are two possible correct answers here.)

(2 pts) Rewrite the integral  $\int x^2 dx$  using a different dummy variable.

(5 pts) What is meant by the phrase 'the linearity of the integral'?

6 pts

Classify each entry as an EXPRESSION or a SENTENCE.

If a *sentence*, state whether it is TRUE, FALSE, or CONDITIONAL.

(2 pts) a)  $\int f(t) dt + \int g(t) dt$

(2 pts) b)  $\int x dx = \frac{1}{2} \int 2x dx$

(2 pts) c)  $f(1) = 2$

32 pts

Evaluate the following indefinite integrals. Be sure to write complete mathematical sentences. Use any appropriate methods.

(8 pts) a)  $\int \left( \frac{2}{x} + e^{3x} - 1 \right) dx$

(8 pts) b)  $\int \ln(x - 1) dx$

(8 pts) c)  $\int \frac{t}{\sqrt[3]{t^2 - 1}} dt$

(8 pts) d)  $\int \frac{3x}{2x + 1} dx$

6 pts

Find a function  $g$  satisfying  $g'(x) = \frac{1}{\sqrt{2x - 1}}$  and  $g(1) = 2$ .

4 pts

Give an antidifferentiation 'counterpart' to the differentiation formula:

$$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$$

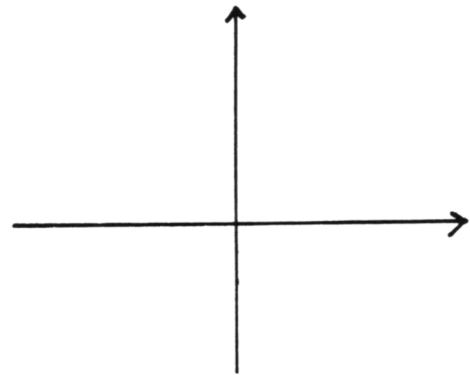
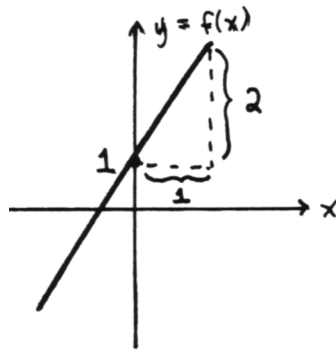
9 pts

(5 pts) Fill in a reason for each step below:

$$\begin{aligned}
 \int (2x + 1)^3 dx &= \int (2x + 1)^3 \cdot \frac{2}{2} dx && ( \quad ) \\
 &= \frac{1}{2} \int (2x + 1)^3 2 dx && ( \quad ) \\
 &= \frac{1}{2} \int u^3 du && ( \quad ) \\
 &= \frac{1}{2} \cdot \frac{u^4}{4} + C && ( \quad ) \\
 &= \frac{1}{8} (2x + 1)^4 + C && ( \quad )
 \end{aligned}$$

(4 pts) Now, CHECK this antidifferentiation problem.

8 pts

A function  $f$  is graphed below. Find all the antiderivatives of  $f$ . Graph two of these antiderivatives in the space provided.

8 pts

A function  $f$  is graphed below. On the same graph, graph another function that has the same derivative as  $f$ , and has a nonremovable discontinuity at  $x = 1$ .