

NAME \_\_\_\_\_

SAMPLE TEST, worth 100 points, Chapters 1 and 2

Show all work that leads to your answers. Good luck!

1. (8 pts) The following symbols are used for important sets of numbers. State the NAME of each set, and GIVE A PRECISE DESCRIPTION of the numbers in each set. The first one is done for you, as a sample.

(0 pt)  $\mathbb{Q}$  is the set of *rational numbers*.  
These are numbers that can be written in the form  $\frac{p}{q}$ , where  $p$  and  $q$  are integers, and  $q \neq 0$ .

(2 pts)  $\mathbb{R}$  \_\_\_\_\_

(2 pts)  $\mathbb{Z}$  \_\_\_\_\_

(2 pts)  $\mathbb{C}$  \_\_\_\_\_

(1 pt) Give an example of an irrational real number. \_\_\_\_\_

(1 pt) Give an example of a positive real number that is not an integer. \_\_\_\_\_

2. Identify the following equations as TRUE, FALSE, or CONDITIONAL:

(12 pts) (1 pt)  $\sqrt{(-3)^2} = -3$  \_\_\_\_\_

(1 pt)  $-4^2 = -16$  \_\_\_\_\_

(1 pt)  $x^3 = x \cdot x \cdot x$  \_\_\_\_\_

(1 pt)  $x = 3$  \_\_\_\_\_

(2 pts) What does it mean to *solve an equation in 1 variable*? (Answer in English.)

(2 pts) How many variables are there in this equation?  $3x + x^2 - 2 = \sqrt{x}$

(2 pts) Solve the equation  $x^2 = 3$ , taking the universal set to be the real numbers.

(1 pt) Solve the equation  $x^2 = 3$ , taking the universal set to be  $\mathbb{Q}$ .

(1 pt) What is the universal set assumed to be in this course, unless otherwise specified?

3. TRUE or FALSE. (Circle the correct response.)

(14 pts) T F For all real numbers  $a$  and  $b$ ,  $(a + b)^2 = a^2 + b^2$ .

(2 pts each) T F The number  $\frac{3}{3 \cdot 2^{57} \cdot 5^{402}}$  has a finite decimal expansion.

T F  $\{x \mid a < x < b\} = \{x \mid a < x \text{ and } x < b\}$

T F  $0 \in (0, 1]$

T F  $\{0\} \subset [0, 1]$

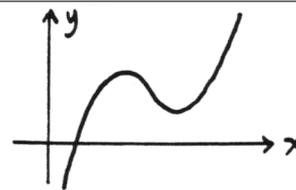
T F  $x = 3 \iff x^2 = 9$

T F The symbol  $\iff$  is used to compare numbers.

4.  
(9 pts)

Consider the graph shown.

(3 pts) Is  $y$  a function of  $x$ ? Why or why not?



(3 pts) Is  $x$  a function of  $y$ ? Why or why not?

(3 pts) Sketch the graph of a function  $f$  satisfying the following properties:  $f$  is one-to-one,  $\mathcal{D}(f) = [-1, 2)$ , and  $\mathcal{R}(f) = [0, 6]$ .

5.  
(12 pts)

In this question, you are asked to sketch several graphs. Put your graphs in the space provided below.

(3 pts) (a) Graph the equation  $x = 5$ , viewed as an equation in 1 variable.

(4 pts) (b) Graph the equation  $x = 5$ , viewed as an equation in 2 variables ( $x$  and  $y$ ).

(5 pts) (c) Graph the equation  $y - 3 = |x - 2|$ .

6.  
(15 pts)

The 'black box' shown corresponds to a function. Please answer the following questions:

(2 pt) What is the NAME of this function? \_\_\_\_\_

(2 pt) What is the output of this function, when the input is  $y$ ? \_\_\_\_\_

(3 pts) Graph this function in the space provided below.



(2 pt) Is this a 1-1 function? (YES or NO) \_\_\_\_\_

(1 pt) What is  $f(-2)$ ? (Write a complete sentence:  $f(-2) = ???$ ) \_\_\_\_\_

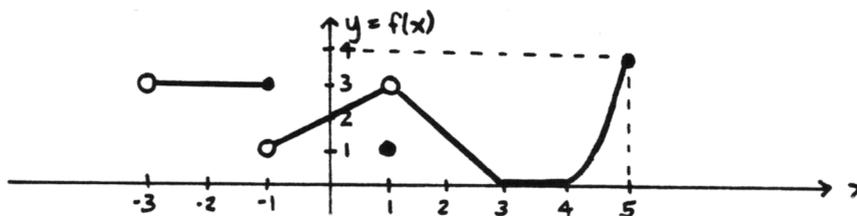
(1 pt) What is  $f(x + h)$ ? \_\_\_\_\_

(2 pt) What is  $\mathcal{D}(f)$ ? \_\_\_\_\_

(2 pts) What is  $\mathcal{R}(f)$ ? \_\_\_\_\_

7. (6 pts) (1 pt) There are two equations that describe the relationship between a 1-1 function  $f$  and its inverse  $f^{-1}$ . One of these looks something like this:  $f(f^{-1}(x)) = x \ \forall x \in \text{????}$ . What must ????? be to get a true statement?
- (2 pts) What is the second equation that describes the relationship between  $f$  and  $f^{-1}$ ?
- (3 pts) The function  $g$  given by  $g(x) = 7x - 2$  is 1-1. Find the inverse function  $g^{-1}$ .

8. (7 pts) The following questions all refer to the function  $f$  whose graph is shown below.



Find the following, if they exist. Be sure to write complete mathematical sentences.

- (2 pts)  $f(1)$  \_\_\_\_\_  $f(3.1)$  \_\_\_\_\_
- (1 pts)  $\mathcal{D}(f)$
- (1 pts)  $\mathcal{R}(f)$
- (2 pts)  $\{x \mid f(x) = 0\}$
- (1 pts)  $\{x \mid f(x) = 4\}$

9. (3 pts) Let  $f$  and  $g$  be functions.
- (2 pts) Define, in the obvious way, a new function named  $f + g$ . Express the formula using the dummy variable  $t$ .
- (1 pt) What is the domain of this new function  $f + g$ ? Answer using a complete mathematical sentence.

10. (4 pts) Solve the equation  $x^3 - 3x + 2 = 0$ . Be sure to write complete mathematical sentences. (HINT: Note that the number 1 is a root of  $x^3 - 3x + 2$ .)

11.  
(3 pts)

(3 pts) Graph the function  $f: (0, 1) \cup (2, 3) \rightarrow \mathbb{R}$ ,  $f(x) = \sqrt{x}$ .

12.  
(7 pts)

(3 pts) Let  $f$  and  $g$  be functions. What is the domain of the composite function  $f \circ g$ ?  
(Write a complete mathematical sentence.)

(4 pts) Consider the function  $g$  defined by the rule  $g(x) = (3x-1)^2$ . This function can naturally be viewed as a composition of three functions. Tell me what each box below does.

