CHAPTER 7
THE DEFINITE INTEGRAL

In the previous chapter, the indefinite integral $\int f(x) \, dx$ was studied. This integral gives all the antiderivatives of the function $f$.

In this chapter, another type of integral is studied, called the definite integral of $f$ on $[a, b]$, and denoted by $\int_a^b f(x) \, dx$. Under suitable conditions, $\int_a^b f(x) \, dx$ gives information about the area trapped between the graph of $f$ and the $x$-axis over the interval $[a, b]$.

The integrals $\int f(x) \, dx$ and $\int_a^b f(x) \, dx$ are, in one sense, very different: $\int f(x) \, dx$ is a class of functions, (all the antiderivatives of $f$), but $\int_a^b f(x) \, dx$ is a number. However, in another sense, the integrals are very much related: the Fundamental Theorem of Integral Calculus tells us that if we know just one antiderivative of $f$, then we can compute the number $\int_a^b f(x) \, dx$. 

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